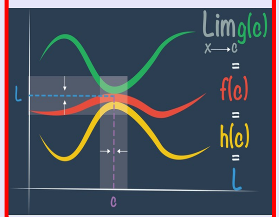
Math 261

Spring 2023

Lecture 54



Feb 19-8:47 AM

Class QZ 15

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Find f_{ave} for $f(x) = \sin 4x$ on $[-\pi, \pi]$.

$$f_{ave} = \frac{1}{\pi-(-\pi)}\int_{-\pi}^{\pi} \sin 4x\,dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} \sin 4x\,dx$$

$u = 4x$

$du = 4dx$

$\frac{du}{4} = dx$

$$= \frac{1}{2\pi}\left[-\frac{1}{4}\cos 4x\right]_{-\pi}^{\pi} = \frac{-1}{8\pi}\left[\cos 4\pi - \cos(-4\pi)\right]$$

$$= \frac{-1}{8\pi}\left[1 - 1\right] = \boxed{0} \checkmark$$

$$\int_{-a}^{a} \text{odd}\,dx = 0$$

$\sin 4x$ is an odd function

$$\int_{-\pi}^{\pi} \sin 4x\,dx = 0$$

May 22-9:40 AM

Class QZ 16

Find f_{ave} for $f(x)=x^2$ on $[-a,a]$.

Exact answer only.

$$f_{ave} = \frac{1}{a-(-a)}\int_{-a}^{a} x^2\,dx = \frac{1}{2a}\int_{-a}^{a} x^2\,dx$$

$$\int_{-a}^{a} \text{even function}\,dx = 2\int_{0}^{a} \text{even}\,dx$$

$$= \frac{1}{\cancel{2}a}\cdot\cancel{2}\int_{0}^{a} x^2\,dx$$

$$= \frac{1}{a}\cdot\frac{x^3}{3}\Big]_{0}^{a}$$

$$= \frac{1}{3a}\cdot a^3 = \boxed{\frac{a^2}{3}}$$

May 23-8:31 AM

$$f(x) = \int_{\sqrt{x}}^{x^2} \frac{t^4}{t^8+1}\,dt$$

Find $f(1)$

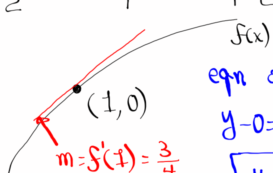
$$f(1) = \int_{\sqrt{1}}^{1^2} \frac{t^4}{t^8+1}\,dt = \int_{1}^{1} \frac{t^4}{t^8+1}\,dt = \boxed{0}$$

$$\int_{a}^{a} f(x)\,dx = \boxed{0}$$

Find $f'(x) = \dfrac{(x^2)^4}{(x^2)^8+1}\cdot 2x - \dfrac{(\sqrt{x})^4}{(\sqrt{x})^8+1}\cdot\dfrac{1}{2\sqrt{x}}$

$$= \frac{2x^9}{x^{16}+1} - \frac{x^2}{(x^4+1)\cdot 2\sqrt{x}}$$

$$f'(1) = \frac{2}{2} - \frac{1}{4} = 1 - \frac{1}{4} = \boxed{\frac{3}{4}}$$



eqn of tan. line

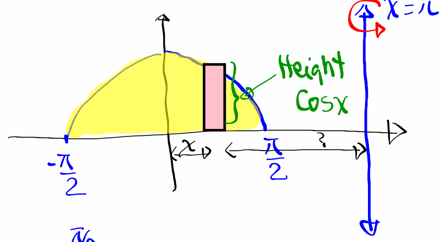
$y - 0 = \frac{3}{4}(x-1)$

$\boxed{y = \frac{3}{4}x - \frac{3}{4}}$

May 23-9:12 AM

Set-up Only:

Find the volume obtained by rotating the region enclosed by $y = \cos x$, $y=0$, $x = -\frac{\pi}{2}$, and $x = \frac{\pi}{2}$ about $x = \pi$.



Ref. Rect. is Parallel to A.O.R.

Shell

$x + ? = \pi$

$? = \pi - x$

$$V = \int_{-\pi/2}^{\pi/2} 2\pi \cdot \boxed{\text{Distance}} \cdot \boxed{\text{Height}}\, dx$$

$$= \boxed{2\pi \int_{-\pi/2}^{\pi/2} (\pi - x) \cdot \cos x \, dx}$$ Set-up

May 23-9:20 AM

Class QZ 17

$$f(x) = \int_2^{\sqrt{x}} \frac{1}{t^4+1}\, dt$$

1) Find $f(4) = \int_2^{\sqrt{4}} \frac{1}{t^4+1} dt = \int_2^{2} \frac{1}{t^4+1} dt = \boxed{0}$

2) Find $f'(x) = \frac{1}{(\sqrt{x})^4+1} \cdot \frac{1}{2\sqrt{x}} - \frac{1}{2^4+1} \cdot 0$

$$= \frac{1}{x^2+1} \cdot \frac{1}{2\sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}(x^2+1)}}$$

3) Find $f'(4) = \frac{1}{2\sqrt{4}(4^2+1)} = \frac{1}{2 \cdot 2(17)} = \boxed{\frac{1}{68}}$

May 23-9:27 AM